Supplementary exercises after tutorial 1

Q1(a). Let $(a_n) \subset \mathbb{R}$ be a bounded sequence. Let $\alpha := \sup_{n\in\mathbb{N}} a_n$. Suppose $\alpha \neq a_k$ for any $k \in \mathbb{N}$. Construct a strictly increasing subsequence (a_{n_k}) such that $\lim_{k\to\infty} a_{n_k} = \alpha$. (Idea: For each $\epsilon > 0$, there are infinitely many $k \in \mathbb{N}$ such that $\alpha - \epsilon < a_k < \alpha$)

(b). If $\sup_{n\geq k} a_n$ is an element of the sequence $(a_n)_{n\geq k}$ for each $k \in \mathbb{N}$, then there is a decreasing subsequence for the sequence (a_n) (may not be strict).

If $\sup_{n\geq k} a_n$ is not an element of $(a_n)_{n\geq k}$ for some k, then from Q1(a), there is a strictly increasing subsequence of (a_n).

Therefore, every sequence admits a monotone subsequence. See our textbook [Bartle] p. 80 **The Existence of Monotone Subsequences** for a clean proof of this statement.